LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2012**

# MT 5408 - GRAPH THEORY

 Date : 27-04-2012 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

**PART A**

**Answer ALL the questions (10 x 2 =20)**

1. Define a complete bipartite graph.
2. Prove that every cubic graph has an even number of points.
3. If *G*1 = *K*2 and *G*2 = *C*3 then find
4. *G*1$∪$*G*2 (ii) *G*1 + *G*2
5. Define distance between any two points of a graph.
6. Define an Eulerian graph and give an example.
7. Prove that every Hamiltonian graph is 2-connected.
8. Draw all possible trees with 6 vertices.
9. Define an eccentricity of a vertex *v* in a connected graph *G*.
10. Is *K*3,3 planar? If not justify your answer.
11. Find the chromatic number for the following graph.



**PART B**

**Answer any FIVE questions (5 x 8 =40)**

1. (a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

(b) Prove that $δ\leq \frac{2q}{p}\leq Δ.$ (**5+3**)

12. If Let $G\_{1}$ be a $\left(p\_{1},q\_{1}\right)$ graph and $G\_{2}$ be a $\left(p\_{2},q\_{2}\right)$ graph then prove that

 (i) $G\_{1}+G\_{2}$ is a $\left(p\_{1}+p\_{2},q\_{1}+q\_{2}+p\_{1}p\_{2}\right)$ graph.

 (ii) $G\_{1}×G\_{2}$ is a $\left(p\_{1}p\_{2},q\_{1}p\_{2}+q\_{2}p\_{1}\right)$ graph.

13. (a) Prove that a closed walk odd length contains a cycle.

 (b) If a graph *G* is not connected then prove that the graph $\overbar{G}$ is connected. (**5+3**)

14. (a) Prove that a line *x* of a connected graph *G* is a bridge if and only if *x* is not on any cycle of *G*.

 (b) Prove that every non - trivial connected graphs has atleast two points which are not cut points. (**5+3**)

15. If *G* is a graph with $p\geq 3$ vertices and $δ\geq ^{p}/\_{2}$, then prove that *G* is Hamiltonian.

16. (a) Prove that every tree has a centre consisting of either one point or two adjacent points.

(b) Let *T* be a spanning tree of a connected graph. Let *x* = *uv* be an edge of *G* not in *T*. Then prove that *T* + *x* contains a cycle. (**4+4**)

17. State and prove Euler’s theorem.

18. Prove that every planar graph is 5-colourable.

**PART C**

**Answer any TWO questions (2 x 20 =40)**

19. (a) Prove that the maximum number of lines among all *p* point graphs with no triangles is $\left[\frac{p^{2}}{4}\right]$.

(b) Let $G$ be a $\left(p,q\right)$ graph then prove that $Γ\left(G\right)=Γ\left(\overbar{G}\right)$. (**15 +5**)

20. (a) Prove that a graph *G* with atleast two points is bipartite if and only if all its cycles are of even length.

(b) Let *G* be a connected graph with atleast three points then prove that *G* is a block if and only if any two points of *G* lie on a common cycle. (**12+8**)

21. (a) Prove that the following statements are equivalent for a connected graph G

(i) *G* is eulerian.

(ii) Every point of *G* has even degree.

(iii) The set of edges of *G* can be partitioned into cycles.

(b) Show that the Petersen graph is nonhamiltonian. (**12+8**)

22. (a) Let $G$ be a $\left(p,q\right)$ graph then prove that the following statements are equivalent

(i) *G* is a tree.

(ii) Every two points of *G* are joined by a unique path.

(iii) *G* is connected and $p=q+1$.

(iv) *G* is acyclic and $p=q+1$.

 (b) Prove that every uniquely *n* – colourable graph is (*n* – 1) connected. (**14+6**)

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